International Journal of Theoretical Physics, Vol. 38, No. 2, 1999

Dirac Quantum Field Theory in Rindler Spacetime

Zhu Jianyang[1] **and Luo Zhijian**[1]

Received January 19, 1998

The dynamical properties of Dirac particles in Rindler spacetime are investigated. It is shown that the vacuum state of the Dirac field in Minkowski spacetime appears to be a thermal state for a Rindler observer, and the usual thermal equilibrium state of the Dirac field in Minkowski spacetime is a quasithermal equilibrium state, which is time independent and characterized by two quasi-temperature parameters for a Rindler observer.

1. INTRODUCTION

When considering the Klein–Gordon scalar field, a vacuum state in Minkowski spacetime appears to be a thermal state for a uniformly accelerated Rindler observer, and the temperature is proportional to the Rindle observer's proper acceleration (Sciama, 1981; Gibbons and Perry, 1978)—the Rindler effect. Unruh gave a simple proof (Unruh, 1976; Birrell and Davis, 1982). Recently, further study showed that, for the Klein–Gordon scalar field, the usual thermal equilibrium state in Minkowski spacetime is no longer the usual thermal equilibrium state for a uniformly accelerated Rindler observer, but is a quasithermal equilibrium state which is time independent and characterized by two quasi-temperature parameters (Zhao *et al.*, 1996). This makes the Rindler effect more complete.

All of the previous work has been done only on the Klein–Gordon scalar field. The study needs to be extended to the Dirac spinor field. In this paper, we will discuss the vacuum state and the thermal equilibrium state of the Dirac field in Minkowski spacetime. By means of second quantization and Bogoliubov transformation on the Dirac spinor field in Rindler spacetime, we get the respective result.

[1] Department of Physics, Jiangxi Normal University, Nanchang 330027, China.

0020-7748/99/0200-057 5$16.00/0

In Section 2, we study the vacuum state of the Dirac field in Minkowski spacetime and show that this state appears to be a thermal state and radiates a Dirac thermal spectrum for a Rindler observer. In Section 3, we study the usual thermal equilibrium state of the Dirac field in Minkowski spacetime and show this state is a quasithermal equilibrium state which is time independent and which is characterized by two quasi-temperature parameters, for the Rindler observer. Section 4 contains a conclusion and discussion.

2. VACUUM STATE IN MINKOWSKI SPACETIME

The line element of two-dimensional Minkowski spacetime is

$$ds^2 = dt^2 - dx^2 \tag{1}$$

Under the coordinate transformation

$$\begin{cases} t = a^{-1}\, e^{a\xi} sha\eta \\ x = a^{-1}\, e^{a\xi} cha\eta \end{cases} \tag{2}$$

(1) is represented as

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) \tag{3}$$

The system (η, ξ) is known as the Rindler coordinate system (Rindler, 1966), and the new spacetime (3) is called Rindler spacetime. Rindler coordinatization of Minkowski space and a conformal diagram of the Rindler system are shown in Figs. 1 and 2. Obviously, the Rindler coordinate system covers only a quadrant of Minkowski space, i.e., the region R. Here L is the mirror region of R, the null rays $t + x = 0$ and $t - x = 0$ are event horizons of regions R and L, the line η = const taken across both R and L is the Cauchy

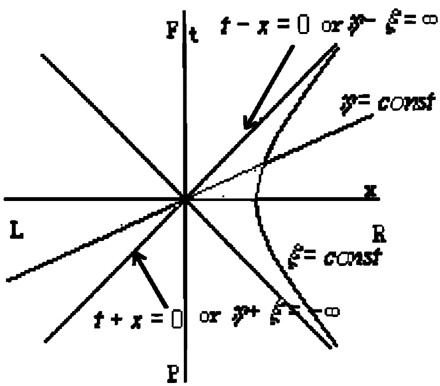

Fig. 1. Rindler coordinatization of Minkowski space.

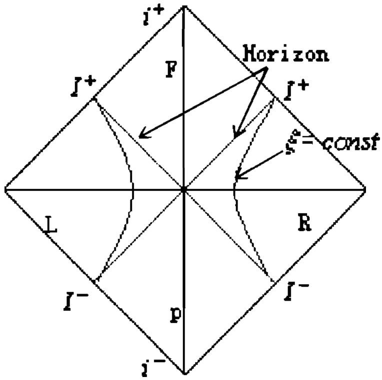

Fig. 2. Conformal diagram of the Rindler system.

surface for the whole spacetime, and F and P are causal future and past regions of $R \cup L$, respectively. In addition, the timelike world lines of Rindler observers (ξ = const) are hyperbolas in the t–x plane ($x^2 - t^2 = a^{-2}e^{2a\xi} =$ constant > 0). This means that the Rindler observer has a uniform acceleration.

In flat spacetime, the dynamical behavior of Dirac particles can be described by the Dirac equation

$$ir^{\alpha}\Psi_{,\alpha} - m\Psi = 0 \tag{4}$$

where r^{α} are Dirac matrices, which satisfy the following anticommutation relations:

$$\{r^{\alpha}, r^{\beta}\} = 2\eta^{\alpha\beta} \tag{5}$$

In an inertial system of Minkowski spacetime, a complete set of mode solutions of the Dirac equation is given as (Lurie, 1968)

$$\frac{1}{\sqrt{V_0}} u_{ks} \exp[i(\mathbf{k} \cdot \mathbf{x} - \omega_k t)] \qquad \text{(positive-frequency modes)} \tag{6}$$

and

$$\frac{1}{\sqrt{V_0}} v_{ks} \exp[-i(\mathbf{k} \cdot \mathbf{x} - \omega_k t)] \qquad \text{(negative-frequency modes)} \tag{7}$$

where

$$u_{ks} = \sqrt{\frac{\omega_k + m}{2\omega_k}} \begin{pmatrix} X_s \\ \dfrac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\omega_k + m} X_s \end{pmatrix}$$

$$v_{ks} = \sqrt{\frac{\omega_k + m}{2\omega_k}} \begin{pmatrix} \dfrac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\omega_k + m} X_s \\ X_s \end{pmatrix}$$

$$X_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad X_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

$s = 1, 2$, correspond to X_+, X_-, respectively, $\mathbf{k}$ is the wave vector, and ω_k is the energy (we use units which $\hbar = c = G = 1$).

Therefore, we can expand the field Ψ as

$$\Psi(\mathbf{x}, t) = \frac{1}{\sqrt{V_0}} \sum_{k,s} (a_{ks} u_{ks} \exp(i\mathbf{kx} - i\omega_k t) + c^+_{ks} v_{ks} \exp(-i\mathbf{kx} + i\omega_k t)) \tag{9}$$

where a_{ks}, c_{ks}, a^+_{ks}, c^+_{ks} are annihilation and creation operators of Dirac particles in Minkowski spacetime, and satisfy the following anticomutation relations:

$$\begin{aligned}
\{a_{ks}, a^+_{k's'}\} &= \delta_{kk'}\delta_{ss'} \\
\{c_{ks}, c^+_{k's'}\} &= \delta_{kk'}\delta_{ss'} \\
\{a_{ks}, a_{k's'}\} &= \{c_{ks}, c_{k's'}\} = 0 \\
\{a^+_{ks}, a^+_{k's'}\} &= \{c^+_{ks}, c^+_{k's'}\} = 0 \\
\{a_{ks}, c_{k's'}\} &= \{a_{ks}, c^+_{k's'}\} = 0 \\
\{c_{ks}, a_{k's'}\} &= \{c_{ks}, a^+_{k's'}\} = 0
\end{aligned} \tag{10}$$

We define the Fock vacuum

$$a_{ks}|0\rangle_M = c_{ks}|0\rangle_M = 0 \qquad \forall k, s \tag{11}$$

$|0\rangle_M$ is called the Minkowski vacuum.

In Rindler spacetime regions R and L, the positive- and negative-frequency mode solutions are given as

$$\begin{cases} {}^R\varphi_{ks} = \begin{cases} \dfrac{1}{\sqrt{V_0}} u_{ks} e^{ik\xi - i\omega\eta} & \text{in} \quad R \\ 0 & \text{in} \quad L \end{cases} \\ {}^R\phi_{ks} = \begin{cases} \dfrac{1}{\sqrt{V_0}} v_{ks} e^{-ik\xi + i\omega\eta} & \text{in} \quad R \\ 0 & \text{in} \quad L \end{cases} \end{cases} \tag{12}$$

$$\begin{cases} {}^L\varphi_{ks} = \begin{cases} \dfrac{1}{\sqrt{V_0}} u_{ks} e^{ik\xi + i\omega\eta} & \text{in} \quad L \\ 0 & \text{in} \quad R \end{cases} \\ {}^L\phi_{ks} = \begin{cases} \dfrac{1}{\sqrt{V_0}} v_{ks} e^{-ik\xi - i\omega\eta} & \text{in} \quad L \\ 0 & \text{in} \quad R \end{cases} \end{cases} \tag{13}$$

The set (12) is complete in the Rindler region R, while (13) is complete in L, but neither set separately is complete on all of Minkowski space. However, both sets together are complete in Minkowski space. The Dirac field may be expanded in ${}^R\varphi_{ks}$, ${}^R\phi_{ks}$, ${}^L\varphi_{ks}$, ${}^L\phi_{ks}$:

$$\Psi(\xi, \eta) = \sum_{k,s}((b_{ks}^{(1)L}\varphi_{ks} + d_{ks}^{(1)+L}\phi_{ks}) + (b_{ks}^{(2)R}\varphi_{ks} + d_{ks}^{(2)+R}\phi_{ks})) \tag{14}$$

where $b_{ks}^{(1,2)}$, $d_{ks}^{(1,2)}$, $b_{ks}^{(1,2)+}$, and $d_{ks}^{(1,2)+}$ are annihilation and creation operators of Dirac particles in Rindler spacetime; they satisfy the anticommutation relations. Define the Fock vacuum

$$b_{ks}^{(1,2)}|0\rangle_{\mathrm{R}} = d_{ks}^{(1,2)}|0\rangle_{\mathrm{R}} = 0 \qquad \forall k,s \tag{15}$$

$|0\rangle_R$ is called the Rindler vacuum.

The functions ${}^R\varphi_{ks}$ and ${}^R\phi_{ks}$ do not go over smoothly to ${}^L\varphi_{ks}$ and ${}^L\phi_{ks}$, respectively, at $\bar{u} = t - x = 0$, $\bar{v} = t + x = 0$ (the crossover point between L and R); hence the Rindler modes, by virtue of their nonanalyticity at $\bar{u} = \bar{v} = 0$, cannot be a combination of pure positive- or negative-frequency Minkowski modes. They must be a mixture of positive and negative frequencies, i.e., $|0\rangle_M \neq |0\rangle_R$.

Imitating Unruh's proof about the Rindler effect, we construct the Unruh mode function. Consider a massless spinor field; we define the Eddington–Finkelstein coordinates

$$\begin{cases} v = \eta + \xi \\ u = \eta - \xi \end{cases} \tag{16}$$

In R, we have

$$\begin{cases} \bar{v} = t + x = a^{-1} e^{av} \\ \bar{u} = t - x = -a^{-1} e^{-au} \end{cases} \tag{17}$$

or

$$\begin{cases} v = \dfrac{1}{a} \, ln(a\bar{v}) \\ u = -\dfrac{1}{a} \, ln(-a\bar{u}) \end{cases} \tag{18}$$

In L, we have

$$\begin{cases} \bar{v} = -a^{-1} e^{av} \\ \bar{u} = a^{-1} e^{-au} \end{cases} \tag{19}$$

or

$$\begin{cases} v = \dfrac{1}{a} \ln(-a\bar{v}) \\ u = -\dfrac{1}{a} \ln(a\bar{u}) \end{cases} \tag{20}$$

Substituting (18) and (20) into (12) and (13), we obtain

$$^{R}\varphi_{ks} = \frac{1}{\sqrt{V_0}} \, u_{ks} e^{i(\omega/a)\ln(-a\bar{u})} \tag{21}$$

$$\begin{aligned} ^{L}\varphi^{*}_{-ks} &= \frac{1}{\sqrt{V_0}} \, u^{*}_{-ks} e^{-i\omega u} = \frac{1}{\sqrt{V_0}} \, u^{*}_{-ks} e^{i(\omega/a)\ln(a\bar{u})} \\ &= \frac{1}{\sqrt{V_0}} \, u^{*}_{-ks} e^{i(\omega/a)[\ln(-a\bar{u})+\ln(-1)]} \\ &= \frac{1}{\sqrt{V_0}} \, u^{*}_{-ks} e^{i(\omega/a)\ln(-a\bar{u})} e^{\pi\omega/a} \end{aligned} \tag{22}$$

$$e^{-\pi\omega/a} \, {}^{L}\varphi^{*}_{-ks} = \frac{1}{\sqrt{V_0}} \, u^{*}_{-ks} e^{i(\omega/a)\ln(-a\bar{u})} \tag{23}$$

We can see that ${}^R\varphi_{ks}$ and $e^{-\pi\omega/a}\ {}^L\varphi^*_{-ks}$ have the same functional form. Similarly, ${}^R\phi_{ks}$ and $e^{-\pi\omega/a}\ {}^L\phi^*_{-ks}$, ${}^L\varphi_{ks}$ and $e^{-\pi\omega/a}\ {}^R\varphi^*_{-ks}$, and ${}^L\phi_{ks}$ and $e^{-\pi\omega/a}\ {}^R\phi^*_{-ks}$ also have the same functional form, so the Unruh mode functions can be constructed as

$$\begin{cases} f^{(1)}_{ks} = e^{\pi\omega/2a}\ {}^R\varphi_{ks} + e^{-\pi\omega/2a}\ {}^L\varphi^*_{-ks} \\ g^{(1)}_{ks} = e^{\pi\omega/2a}\ {}^R\phi_{ks} + e^{-\pi\omega/2a}\ {}^L\phi^*_{-ks} \end{cases} \tag{24}$$

$$\begin{cases} f^{(2)}_{ks} = e^{\pi\omega/2a}\ {}^L\varphi_{ks} + e^{-\pi\omega/2a}\ {}^R\varphi^*_{-ks} \\ g^{(2)}_{ks} = e^{\pi\omega/2a}\ {}^L\phi_{ks} + e^{-\pi\omega/2a}\ {}^R\phi^*_{-ks} \end{cases} \tag{25}$$

$f^{(1,2)}_{ks}$ and $g^{(1,2)}_{ks}$ share the analyticity properties of Minkowski positive- and negative-frequency modes, and must also share a common vacuum state $|0\rangle_M$; thus we can expand Ψ in terms of $f^{(1,2)}_{ks}$ and $g^{(1,2)}_{ks}$:

$$\Psi(\xi, \eta) = \sum_{k,s} [2ch(\pi\omega/a)]^{-1/2}\ (B^{(1)}_{ks} f^{(1)}_{ks} + D^{(1)+}_{ks} g^{(1)}_{ks} + B^{(2)}_{ks} f^{(2)}_{ks} + D^{(2)+}_{ks} g^{(2)}_{ks}) \tag{26}$$

where $B^{(1,2)}_{ks}$, $D^{(1,2)}_{ks}$, $B^{(1,2)+}_{ks}$, and $D^{(1,2)+}_{ks}$ are annihilation and creation operators of Dirac particles in Minkowski spacetime when we adopt Rindler coordinates. They satisfy the anticommutation relations, and the annihilation operators satisfy

$$B^{(1,2)}_{ks}|0\rangle_M = D^{(1,2)}_{ks}|0\rangle_M = 0 \qquad \forall k, s \tag{27}$$

Equations (24) and (25) are not normalized; in (26) we introduce the normalization constant. The inner product on the Dirac field in Minkowski spacetime is defined as

$$(\psi, \phi) = \int \psi^+\phi\ d^3x \tag{28}$$

In two-dimensional Rindler spacetime, (28) can be written as

$$(\psi, \phi) = \int \psi^+\phi\ d\xi \tag{29}$$

From (29), we have

$$({}^R\varphi_{ks}, {}^R\varphi_{k's'}) = ({}^R\phi_{ks}, {}^R\phi_{k's'}) = ({}^L\varphi_{ks}, {}^L\varphi_{k's'}) = ({}^L\phi_{ks}, {}^L\phi_{k's'}) = \delta_{kk'}\delta_{ss'}$$

$$({}^R\varphi^*_{ks}, {}^R\varphi^*_{k's'}) = ({}^R\phi^*_{ks}, {}^R\phi^*_{k's'}) = ({}^L\varphi^*_{ks}, {}^L\varphi^*_{k's'}) = ({}^L\phi^*_{ks}, {}^L\phi^*_{k's'}) = \delta_{kk'}\delta_{ss'}$$

$$({}^R\varphi_{ks}, {}^R\phi^*_{k's'}) = ({}^R\phi_{ks}, {}^R\varphi^*_{k's'}) = ({}^L\varphi_{ks}, {}^L\phi^*_{k's'}) = ({}^L\phi_{ks}, {}^L\varphi^*_{k's'}) = \delta_{kk'}\delta_{ss'}$$

$$({}^R\varphi_{ks}, {}^R\varphi^*_{k's'}) = ({}^R\phi_{ks}, {}^R\phi^*_{k's'}) = ({}^L\varphi_{ks}, {}^L\varphi^*_{k's'}) = ({}^L\phi_{ks}, {}^L\phi^*_{k's'}) = 0$$

$$({}^R\varphi_{ks}, {}^R\phi_{k's'}) = ({}^R\phi^*_{ks}, {}^R\varphi^*_{k's'}) = ({}^L\varphi_{ks}, {}^L\phi_{k's'}) = ({}^L\phi^*_{ks}, {}^L\varphi^*_{k's'}) = 0 \tag{30}$$

By taking the inner products $(\Psi, {}^L\varphi_{ks})$, $(\Psi, {}^R\varphi_{ks})$, $(\Psi, {}^L\phi_{ks})$, $(\Psi, {}^R\phi_{ks})$, first with Ψ given by (14), and then by (26), we obtain the Bogoliubov transformations

$$\begin{cases} b^{(1)}_{ks} = [2ch(\pi\omega/a)]^{-1/2}\,[e^{\pi\omega/2a}\,B^{(2)}_{ks} + e^{-\pi\omega/2a}\,D^{(1)+}_{-ks}] \\ b^{(2)}_{ks} = [2ch(\pi\omega/a)]^{-1/2}\,[e^{\pi\omega/2a}\,B^{(1)}_{ks} + e^{-\pi\omega/2a}\,D^{(2)+}_{-ks}] \\ d^{(1)}_{ks} = [2ch(\pi\omega/a)]^{-1/2}\,[e^{\pi\omega/2a}\,D^{(2)}_{ks} + e^{-\pi\omega/2a}\,B^{(1)+}_{-ks}] \\ d^{(2)}_{ks} = [2ch(\pi\omega/a)]^{-1/2}\,[e^{\pi\omega/2a}\,D^{(1)}_{ks} + e^{-\pi\omega/2a}\,B^{(2)+}_{-ks}] \end{cases} \tag{31}$$

By means of (31) we have

$$\begin{cases} {}_M\langle 0\big| b^{(1,2)+}_{ks}\, b^{(1,2)}_{ks}\big|0\rangle_M = (e^{2\pi\omega/a} + 1)^{-1} \\ {}_M\langle 0\big| d^{(1,2)+}_{ks}\, d^{(1,2)}_{ks}\big|0\rangle_M = (e^{2\pi\omega/a} + 1)^{-1} \end{cases} \tag{32}$$

If we define temperature $T_0 = a/2\pi K_B$, (32) is represented as

$$\begin{cases} {}_M\langle 0\big| b^{(1,2)+}_{ks}\, b^{(1,2)}_{ks}\big|0\rangle_M = (e^{\omega/K_BT_0} + 1)^{-1} \\ {}_M\langle 0\big| d^{(1,2)+}_{ks}\, d^{(1,2)}_{ks}\big|0\rangle_M = (e^{\omega/K_BT_0} + 1)^{-1} \end{cases} \tag{33}$$

This is precisely the Dirac thermal spectrum for radiation, where T_0 is the coordinate temperature, a is the coordinate acceleration of the Rindler observer, and K_B is Boltzmann's constant.

We can see that the vacuum state of the Dirac field in Minkowski spacetime appears to be a thermal state and radiates a Dirac thermal spectrum for a Rindler observer.

3. THERMAL EQUILIBRIUM STATE IN MINKOWSKI SPACETIME

Now, we further study what a uniformly accelerated observer in the Rindler regions R and L will see when there exists a thermal equilibrium state of the Dirac field in Minkowski spacetime. The state function $|\Psi\rangle_M$ and Hamiltonian H are

$$\begin{aligned} \big|\Psi\rangle_M &= \big|n^{(1)}_{1s}\, n^{(1)}_{2s}\,\ldots,\, n^{(2)}_{1s}\, n^{(2)}_{2s}\,\ldots,\, m^{(1)}_{1s}\, m^{(1)}_{2s}\,\ldots,\, m^{(2)}_{1s}\, m^{(2)}_{2s}\,\ldots\rangle_M \\ &= \prod_{k,s} (B^{(1)+}_{ks})^{n^{(1)}_{ks}}\,(B^{(1)+}_{ks})^{n^{(2)}_{ks}}\,(D^{(1)+}_{ks})^{m^{(1)}_{ks}}\,(D^{(2)+}_{ks})^{m^{(1)}_{ks}}\,\big|0\rangle_M \end{aligned} \tag{34}$$

$$\mathrm{H} = \sum_{k,s} \omega_k[(B^{(1)+}_{ks}\,B^{(1)}_{ks} + D^{(1)+}_{ks}D^{(1)}_{ks}) + (B^{(2)+}_{ks}\,B^{(2)}_{ks} + D^{(2)+}_{ks}D^{(2)}_{ks})] \tag{35}$$

The Hamiltonian H is obtained from

$$H = i \int \Psi^{+} \partial_0 \Psi \, dx^3 \tag{36}$$

As we use two-dimensional Rindler coordinates, we can write (36) as

$$H = i \int \Psi^{+} \partial_\eta \Psi \, d\xi \tag{37}$$

Note that L is the mirror region of R. The Killing vector in R is $+\partial\eta$, but in L it is $-\partial\eta$, so in R, $E_K = \hbar\omega_k = \omega_k = |\mathbf{k}|$, and in L, $E_K = \omega_k = -|\mathbf{k}|$. In (32) we define $\omega_k = |\mathbf{k}|$.

Taking the thermal equilibrium state as a canonical ensemble, we can give the density operator as

$$\hat{\rho} = e^{(F-H)/K_B T} = e^{\beta(F-H)}, \qquad \beta = 1/K_B T \tag{38}$$

where F is the Helmholtz free energy, and H is the system Hamiltonian. From

$$\mathrm{tr}\, \hat{\rho} = 1 \tag{39}$$

we have

$$e^{-\beta F} = \mathrm{tr}\, e^{-\beta H} \tag{40}$$

By mean of (31), (34), and (35), we obtain

$$\begin{aligned}
\langle b_{ks}^{(1,2)+} b_{ks}^{(1,2)} \rangle_{M,\beta} &= \mathrm{tr}(\hat{\rho} b_{ks}^{(1,2)+} b_{ks}^{(1,2)}) \\
&= \mathrm{tr}(e^{-\beta H} b_{ks}^{(1,2)+} b_{ks}^{(1,2)})/\mathrm{tr}(e^{-\beta H}) \\
&= (e^{\beta\omega_k} + e^{2\pi\omega_k/a})/(e^{\beta\omega_k} + 1)(e^{2\pi\omega_k/a} + 1) \\
\langle d_{ks}^{(1,2)+} d_{ks}^{(1,2)} \rangle_{M,\beta} &= \mathrm{tr}(\hat{\rho} d_{ks}^{(1,2)+} d_{ks}^{(1,2)}) \\
&= \mathrm{tr}(e^{-\beta H} d_{ks}^{(1,2)+} d_{ks}^{(1,2)})/\mathrm{tr}(e^{-\beta H}) \\
&= (e^{\beta\omega_k} + e^{2\pi\omega_k/a})/(e^{\beta\omega_k} + 1)(e^{2\pi\omega_k/a} + 1)
\end{aligned} \tag{41}$$

The usual thermal equilibrium state of the Dirac field in Minkowski spacetime is a quasithermal equilibrium state which is time independent and characterized by two quasi-temperature parameters for a Rindler observer.

Equation (41) goes over to the ordinary Minkowski thermal equilibrium state when the acceleration a of the Rindler observer tends to zero,

$$\begin{cases} \langle b_{ks}^{(1,2)+} b_{ks}^{(1,2)} \rangle_{M,\beta} \to (e^{\beta\omega_k} + 1)^{-1} \\ \langle d_{ks}^{(1,2)+} d_{ks}^{(1,2)} \rangle_{M,\beta} \to (e^{\beta\omega_k} + 1)^{-1} \end{cases} \tag{42}$$

On the other hand, when the temperature of the thermal state in the Minkowski spacetime goes to zero, the Rindler observer will see an ordinary Rindler effect,

$$\begin{cases} \langle b_{ks}^{(1,2)+} b_{ks}^{(1,2)} \rangle_{M,\beta} \to (e^{2\pi\omega/a} + 1)^{-1} \\ \langle d_{ks}^{(1,2)+} d_{ks}^{(1,2)} \rangle_{M,\beta} \to (e^{2\pi\omega/a} + 1)^{-1} \end{cases} \tag{43}$$

This is (32).

4. CONCLUSION AND DISCUSSION

The vacuum state of the Dirac field in Minkowski spacetime appears to be a thermal state and radiates a Dirac thermal spectrum for a Rindler observer. The usual thermal equilibrium state of the Dirac field in Minkowski spacetime is a quasithermal equilibrium state, which is time independent and characterized by two quasi-temperature parameters for a Rindler observer.

We have extended the Rindler effect to the Dirac spinor field. By means of second quantization and the Bogoliubov transformation on the Dirac spinor field in Rindler spacetime, we obtain a similar result to the Klein–Gordon scalar field. This is what we expect. But it is noteworthy that the Dirac field is much more complex than the Klein–Gordon field; the Dirac equation possesses not only positive-, but also negative-frequency mode solutions, so the quantization of the Dirac field is different from the Klein–Gordon field, and the study of the Rindler effect is more complicated.

REFERENCES

Birrell, N. D., and Davis, P. C. W. (1982). *Quantum Fields in Curved Space*. Cambridge University Press, Cambridge.

Gibbons, G. W., and Perry, M. J. (1978). *Proc. R. Soc. Lond. A*, **358**, 467.

Lurie, D. (1986). *Particles and Field*, Wiley, New York.

Rindler, W. (1966). *Am. J. Phys.* **34**, 1174.

Sciama, D. W. (1981). *Adv. Phys.* **30**, 327.

Unruh, W. G. (1976). *Phys. Rev. D*, **14**, 870.

Zhao Zheng, Zhu Jian-yang, and Misra, B. (1996). *Int. J. Theor. Phys.* **35**, 741.